Cognitive mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing and their rela....

2 authors:

Gavin Price
Vanderbilt University
26 PUBLICATIONS **829** CITATIONS

Eric D. Wilkey
Vanderbilt University
10 PUBLICATIONS **27** CITATIONS

Cognitive mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing and their relation to math

Gavin R. Price*, Eric D. Wilkey

Department of Psychology & Human Development, Peabody College, Vanderbilt University, 230 Appleton Place, Nashville, TN, 37203, United States

ABSTRACT

Recent studies suggest that the relation between nonsymbolic magnitude processing skills and math competence is mediated by symbolic number processing. However, less is known about whether mapping between nonsymbolic and symbolic magnitude representations also mediates that relation, and whether the mediating role of symbolic number processing is explained by domain-general executive functions. Therefore, the current study examines whether symbolic comparison, mixed-format comparison, and executive function each mediate the relation between nonsymbolic magnitude processing and math. Furthermore, we investigate whether the relation between nonsymbolic and symbolic magnitude comparison is mediated by mapping between the formats and/or domain-general executive functions. Results indicate that symbolic processing mediates the relation between nonsymbolic processing and math, even after controlling for multiple components of executive function, which were also significant mediators. Cross-format comparison (i.e. mapping), on the other hand, did not mediate the relation between nonsymbolic comparison and math, but did mediate the relation between nonsymbolic and symbolic magnitude processing, even after controlling for executive function, which also mediated that relation. Taken together, our results suggest that both domain-specific and domain-general cognitive mechanisms account for the link between nonsymbolic and symbolic magnitude processing and their relation to math.

1. Introduction

Mathematical competence is an important predictor of success in modern life, including educational achievement, employment, financial stability, and physical and mental health (Bynner & Parsons, 1997; Gross, Hudson, & Price, 2009; Parsons & Bynner, 2005). However, a large number of individuals fail to acquire the math skills necessary to function optimally in today's society (Gross et al., 2009; NCES, 2007). Over the past decade, a growing body of research has elucidated important links between basic numerical processing abilities and the development of school level mathematical skills. In particular, it has been suggested that the ability to efficiently process numerical magnitude information in both nonsymbolic (e.g. sets of dots) and symbolic (e.g. Arabic digits) formats is an important foundational competence for math development (for a review see De Smedt, Noël, Gilmore, & Ansari, 2013). However, the extent to which they scaffold math development independently of one another, and independently of domain-general cognitive mechanisms, such as executive function, remains unclear. The current study addresses this uncertainty by investigating the interrelations between nonsymbolic and symbolic magnitude processing and executive function as they relate to math competence in

---

* Corresponding author.
  E-mail address: gavin.price@vanderbilt.edu (G.R. Price).

a large sample of middle-school children. Also unclear are the cognitive mechanisms which support the mapping between nonsymbolic and symbolic representations of numerical magnitude. The second aim of this study, therefore, is to investigate the role of domain-specific and domain-general cognitive processes in the relation between nonsymbolic and symbolic representations of numerical magnitude.

1.1. Mechanisms underlying the relation between numerical magnitude processing and math

Nonsymbolic magnitude processing is typically measured using tasks that require participants to judge which of two sets of dots or other objects contains more items. Performance on this task has been suggested to reflect the precision of the so-called 'approximate number system' (ANS) (Feigenson, Dehaene, & Spelke, 2004). Nonsymbolic magnitude comparison performance has been shown to predict math competence in typically developing children and adults (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Odic, & Halberda, 2012; Mazzocco, Feigenson, & Halberda, 2011a,b) and to be impaired in children with mathematical learning difficulties (Mazzocco et al., 2011a,b; Piazza et al., 2010). It should also be noted, however, that a number of studies have tested for and not observed a significant relation between nonsymbolic magnitude comparison and math performance in both children and adults (e.g. Holloway & Ansari, 2009; Mundy & Gilmore, 2009; Price, Palmer, Battista, & Ansari, 2012). At the same time, a number of studies have reported significant relations between symbolic magnitude comparison tasks, in which participants compare the relative numerical size of two Arabic digits, and math competence (e.g. Bugden & Ansari, 2011; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009). However, again it should be noted that some studies have tested for and not observed any such relation (e.g. Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Thus, the exact pattern of relations between these basic competencies and math outcomes are not yet fully resolved.

Therefore, as an alternative to contrasting the independent relations between nonsymbolic and symbolic magnitude processing and math competence, it may be fruitful to consider the interplay between them and how that interplay relates to math. Specifically, recent evidence suggests that the relation between nonsymbolic magnitude processing and math may be mediated by symbolic number processing (Fazio, Bailey, Thompson, & Siegler, 2014; Lyons & Beilock, 2011; Price & Fuchs, 2016), numeral knowledge (Peng, Yang, & Meng, 2017) and 'number-numerosity mapping' as indexed by dot set estimation (Wong, Ho, & Tang, 2016). According to these studies, nonsymbolic magnitude processing may influence math outcomes by facilitating the acquisition of numerical symbols, which in turn influences the acquisition of basic math skills. Additionally, recent studies have shown that the relation between nonsymbolic magnitude processing and math performance is non-significant when controlling for domain-general factors such as inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013). Thus, both domain-general factors and domain-specific mechanisms have been suggested to influence the relation between numerical magnitude processing and math. While previous mediation studies have controlled for working memory and inhibitory control in their models, it is unclear whether those cognitive processes also serve as mediators of the relation between nonsymbolic magnitude processing and math.

Therefore, in the current study we take multiple approaches to investigate the factors underlying the relation between magnitude processing mechanisms and math. First, we examine whether symbolic comparison and different components of executive function each mediate the relation between nonsymbolic magnitude processing and math, and whether any mediating role of symbolic number processing is accounted for by executive function. Second, we test the mediating role of cross-format magnitude comparison (i.e. comparing the magnitude of a set of dots to an Arabic digit) as a measure of the mapping between nonsymbolic and symbolic representations. This allows us to further test the hypothesis that it is the *mapping* between nonsymbolic and symbolic number representations that scaffolds the acquisition of math skills. Lastly, a growing body of evidence also suggests that there may be a bidirectional influence between nonsymbolic and symbolic magnitude processing whereby the acquisition of numerical symbols refines the representation of nonsymbolic magnitude (Mussolin, Nys, Leybaert, & Content, 2015; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). Therefore, we also examine whether nonsymbolic comparison, mixed-format comparison, and executive function each mediate the relation between symbolic magnitude processing and math.

1.2. Mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing

The apparent importance of the relation between nonsymbolic and symbolic magnitude processing and math development gives rise to a second important question. Specifically, what are the cognitive mechanisms underlying the relation between Arabic digits and the quantities they represent? The most prominent current theory, the 'mapping hypothesis', suggests that Arabic digits are associated with or 'mapped onto' the innate ANS over the course of learning (Dehaene, 2007; Piazza, 2011; for a review see Leibovich & Ansari, 2016). Evidence for this theory comes largely from the fact that across studies, number comparison tasks using both nonsymbolic and symbolic stimuli demonstrate numerical ratio effects, whereby comparison performance declines as the ratio of the larger to the smaller number increases (for a review see: Mussolin et al., 2015). However, the extent to which symbolic numbers are rooted in an underlying representation of nonsymbolic numerical magnitude is still an open empirical question (Matejko & Ansari, 2016; Leibovich & Ansari, 2016).

An alternative explanation may be that the overlap in performance profiles is accounted for by shared domain-general cognitive resources used for comparing the magnitudes of both nonsymbolic and symbolic numbers. The most likely mechanisms in our opinion are executive function, including inhibitory control, task switching, and working memory, all of which are known to play an important role in math development (e.g. Blair, Knipe, & Gamson, 2008; Blair & Razza, 2007). Therefore, in the current study we examine whether the relation between nonsymbolic and symbolic numerical magnitude comparison is mediated by performance on a mixed-format magnitude comparison task, as a measure of the mapping between symbolic and nonsymbolic numbers, and/or by

measures of executive function. We also examine whether any mediating role of mixed-format comparison performance persists while controlling for performance on executive function measures. According to the mapping hypothesis, if symbolic numbers are grounded in nonsymbolic magnitude representations, then performance on the mixed-format comparison task is expected to mediate the relation between them. According to the domain-general hypothesis, on the other hand, if the association between numerical formats is not a result of shared underlying representations, then any mediating role of mixed-format comparison performance should be accounted for by executive function.

In summary, previous literature shows that the relation between nonsymbolic magnitude processing and math competence is mediated by symbolic number processing skills, even after controlling for the domain-general mechanisms of inhibitory control, task switching, and working memory. However, it is unclear whether (1) those domain-general factors also serve as mediators of the relation between nonsymbolic magnitude processing and math competence, (2) the relation depends on a shared representation, or *mapping*, with symbolic magnitudes, and (3) any such relations are bi-directional with symbolic magnitudes. Secondly, the mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing are not fully known. The present study addresses these gaps in the literature by investigating the mediating relations between nonsymbolic, symbolic, and cross-format magnitude comparison and math competence in a large sample of middle-school children.

2. Methods

2.1. Participants

The current sample was drawn from an ongoing longitudinal study of students who participated in an earlier, short-term longitudinal study of early math skills (Pre-K to 1st grade) (Hofer, Lipsey, Dong, & Farran, 2013). The final analytic sample for the original scale-up study included 771 children. In the follow-up study, we were able to locate 628 students who were attending public school in the 2013–14 year in the same district as they attended in Pre-K (16 had withdrawn from the study in 1st grade and were not contacted for further participation, 29 had moved out of the state, 53 had moved out of the district, and 45 were not located despite all efforts). Of those 628, we were able to obtain parental consent and assess 506 children in the 2014–2015 school year. Our final sample was comprised of 475 students for whom we had complete measures from Pre-K to 6th grade (264 females). Of our 475 students who should have been in 6th grade in the 2014–15 school year if they had not been retained or promoted early, 75 (16%) were still in 5th grade and 1 (0.2%) had been promoted to 7th grade. The sample students were located in 76 schools in the first year of the follow-up study, including 31 elementary schools, 27 middle schools, 11 charter schools, and 7 Innovation Cluster schools (schools that had been targeted for additional resources to boost low student achievement). Family income level was inferred on the basis of whether participants qualified for free or reduced lunches (family income less than 1.85 times the U.S. Federal income poverty guideline). In the current sample 94% of participants qualified for free and reduced lunches.

2.2. Procedure

All students were consented to participate and the study was approved by the Vanderbilt University IRB. Assessments were conducted by trained members of the research staff of the Peabody Research Institute. The number comparison tasks, cognitive measures, and math achievement measures were administered during the Spring semester of the students' 6th grade year, given that they had not been held back or promoted early. The number comparison tasks and cognitive measures were administered via tablet computers. All testing was completed in a quiet location at the students' school with one-to-one assistance from trained staff.

2.3. Number comparison tasks

2.3.1. Nonsymbolic number comparison

Participants were presented with two sets of dots simultaneously and asked to indicate via button press which set was more numerous (i.e., which set contained more dots). The set on the left side of the screen contained yellow dots and the set on the right side contained blue dots, which corresponded to color-coded left and right buttons (Fig. 1, A). Response side were fully counterbalanced. Trials consisted of 1200 ms stimulus presentation followed by 1800 ms of a fixation cross. Seven ratios were presented, ranging from 0.33 (5 vs. 15) to 0.9 (9 vs. 10). The number of dots in each stimulus ranged from 5 to 15. Each ratio was presented 10 times for a total of 70 trials. Ratios, stimulus presentation times, and order of presentation were modeled after Odic, Hock, and Halberda (2014). To control for the possibility that participants might choose a strategy based on visual cues rather than number of dots, the following visual properties of dot sets were varied using a modified version of the MATLAB code recommended by Gebuis & Reynvoet (Gebuis & Reynvoet, 2011) to generate stimuli: convex hull (area extended by a stimulus), total surface area (aggregate value of dot surfaces), average dot diameter, and density (convex hull divided by total surface area). In approximately one quarter of the trials all four visual properties were congruent with greater numerosity (i.e. the greater number of dots had a greater convex hull, surface area, etc.). In another approximate quarter of the trials, all four visual properties were incongruent with greater numerosity. In the remaining trials, visual properties were mixed congruent and incongruent.

2.3.2. Symbolic number comparison

Participants were simultaneously presented with two, double-digit Arabic numerals and asked to indicate via button press which of the two was numerically larger (e.g., 54 is larger than 18)(Fig. 1, B). The ratios presented, order of ratios, and stimuli durations
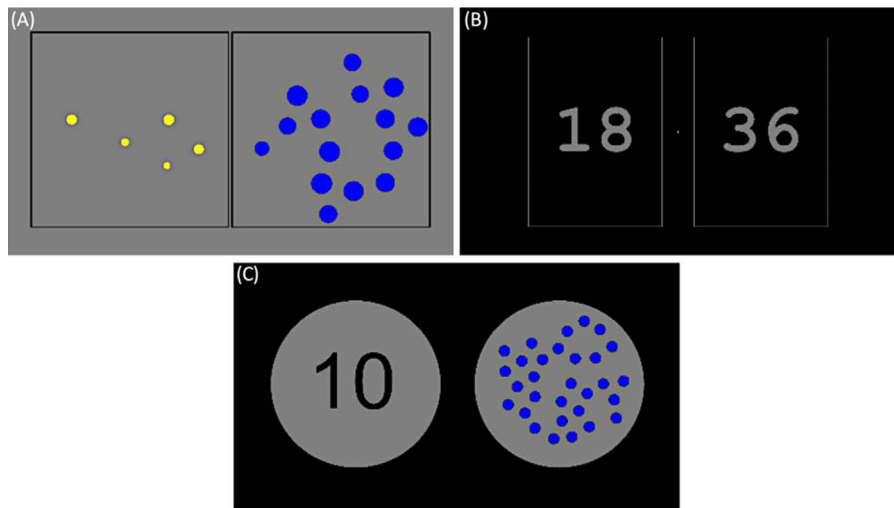
Fig. 1. Example screenshots for (A) nonsymbolic, (B) symbolic, and (C) mixed-format magnitude comparison tasks.

were identical to those in the nonsymbolic number comparison task. To prevent responses uniquely based on the rightmost digit (unit value), the unit–decade compatibility was manipulated such that all trials were decade-incompatible. In other words, the larger number of the pair always had a larger decade but a smaller unit than the smaller number (e.g. 54 vs. 18, 72 vs. 63).

2.3.3. Mixed-format comparison

Similar to both of the above tasks, participants were presented with two simultaneously presented stimuli, one set of dots and one double-digit Arabic numeral, and asked to indicate via button press which of the two dots was numerically larger (e.g. "12" vs 24 dots)(Fig. 1, C). The ratios presented, order of ratios, and stimuli durations were identical to those in the other number comparison tasks. Dot stimuli ranged from 10 to 30 dots per dot set in order to ensure that individuals did not have time to count, given that there was only one set of dots in the mixed-format comparison condition. Both Arabic digits and dot arrays were presented within two grey circles presented on a black background. Arabic digits were presented in black and dots were presented in blue. All of the dots were the same size for the mixed-format comparison task.

2.3.4. Number comparison task performance metrics

A growing body of literature suggests that mean accuracy is highly correlated with and possibly more reliable than ratio dependent metrics such as the *weber fraction* (Gilmore, Attridge, & Inglis, 2011; Inglis & Gilmore, 2014), and that ratio effects are not equivalent across formats (Lyons, Nuerk, & Ansari, 2015). Therefore, in the current study mean accuracy percentages were used to index performance on each of our number comparison tasks.

2.4. Executive function measures

As inhibitory control, working memory, and task switching have been identified as separable components of executive function (Miyake et al., 2000), the current study measures these three cognitive constructs in two tasks. Inhibitory control and task shifting are measured via the Hearts and Flowers task and working memory is measured via the backward Corsi block tapping task. In the current paper, for the sake of clarity we refer to them collectively as measures of executive function and independently as the cognitive constructs they represent. However, in mediation models and other tables, the measures are referred to by the task names themselves in order to be clear about what measures were included in the model.

2.4.1. Inhibitory control and task switching

The Hearts and Flowers task (Wright & Diamond, 2014) was used as measure of students' task switching and inhibitory control. In this task, the child was first presented with a heart on either side of the screen and instructed to press the button that corresponds to the side of the screen with the heart. This first block comprised 12 trials. In the second block of trials (also 12 trials), the child was presented with flowers and asked to press the button that is opposite the side of the flower. In the third set of trials, the child was randomly presented with both hearts and flowers and asked to follow the rule that corresponds to hearts and flowers respectively. The third block comprised 48 trials. To index inhibitory control and task switching we used mean accuracy from the mixed-condition run, and as such, our measure captures both task switching and inhibitory control (Diamond, 2014).

2.4.2. Working memory

The backward Corsi block-tapping test (Corsi, 1972) provided a measure of visuo-spatial working memory. In this computerized task, children first viewed squares light up in a sequence on the screen, and then the student were asked to tap the squares in the

reverse order from which they lit up. The task consists of 16 total possible trials, including two practice trials. The student was given 2 attempts to correctly repeat the reverse sequence per sequence length. The sequence length of squares increased from 2 to 8 across the activity. If the student correctly answered at least 1 of the 2 attempts correctly, the student then proceeded on to the longer (more difficult) sequence. The score of interest was the highest span with a correctly repeated sequence.

2.5. Math achievement measures

2.5.1. KeyMath 3

The KeyMath 3 Diagnostic Assessment (Connolly, 2007) is a comprehensive, norm-referenced measure of essential mathematical concepts and skills. We used three subscales out of the five subscales in the Basic Concepts area. (1) Numeration: The Numeration subtest measures an individual's understanding of whole and rational numbers. It covers topics such as identifying, representing, comparing, and rounding one-, two-, and three-digit numbers as well as fractions, decimal values, and percentages. It also covers advanced numeration concepts such as exponents, scientific notation, and square roots. (2) Algebra: The Algebra subtest measures an individual's understanding of pre-algebraic and algebraic concepts. It covers topics such as sorting, classifying, and ordering by a variety of attributes; recognizing and describing patterns and functions; working with number sentences, operational properties, variables, expressions, equations, proportions, and functions; and representing mathematical relationships. (3) Geometry: The Geometry subtest measures an individual's ability to analyze, describe, compare, and classify two-and three-dimensional shapes. It also covers topics such as spatial relationships and reasoning, coordinates, symmetry, and geometric modeling. Scale scores in the KeyMath 3 are age-normed to reflect population means of 10 and a standard deviation of 3 for each subtest. Math competence was indexed using a composite score (KeyMath 3 Composite) calculated as the mean of the age-scaled scores of the three KeyMath 3 subtests administered, so as to capture performance in a wider range of math skills.

3. Results

3.1. Relations between numerical magnitude processing, executive function, and math

Descriptive statistics of number comparison task performance, executive function measures, and math achievement measures are reported in Table 1. Bivariate correlations between each of the administered measures, as well as the KeyMath 3 composite score are reported in Table 2. To correct for multiple comparisons, the critical *p*-values for each set of correlations were adjusted using the Benjamini-Hochberg's (B-H) False Discovery Rate method with *Q* (false discovery rate) = 0.05 (Benjamini & Hochberg, 1995), which provides a good balance between controlling for false positives and power for detecting weaker, but significant relationships (Supplementary Table S1). All correlations remained significant after correction.

3.2. Mediating relations between numerical magnitude processing and math

3.2.1. Mediators of the relation between nonsymbolic magnitude processing and math

To assess the mediating effect of symbolic magnitude processing and executive function measures (Hearts & Flowers and Backward Corsi) on the relation between nonsymbolic magnitude processing and math competence, we conducted a simple mediation model using the PROCESS Macro in SPSS (Hayes, 2013). To test for significant indirect effects, we used bootstrapping with 5000 resamples to obtain bias-corrected 95% confidence intervals. If zero is outside the confidence intervals, the indirect effect is consequently not zero and can thus be interpreted as evidence of mediation (Preacher & Hayes, 2008). The confidence intervals resulting from this analysis did not contain zero for any of the mediators (Fig. 2), suggesting that symbolic magnitude comparison performance, inhibitory control/task switching, and working memory each mediate the relation between nonsymbolic magnitude comparison and math competence. In contrast, performance on the mixed-format comparison task was not a significant mediator, indicating further that the relation is not dependent on efficiency of mapping between symbolic and nonsymbolic magnitude systems, so far as this relation is captured by the current task.

Table 1
Descriptive statistics for experimental and standardized measures.

| N = 475 (267 females)	Mean	SD	Range
Age (years)	12.1	0.3	11.4–13.4
Nonsymbolic accuracy (%)	75.2	5.3	58.6–91.4
Symbolic accuracy (%)	91.2	6.6	63.0–100
Mixed-format accuracy (%)	69.9	7.5	53.0–93.0
Backward Corsi (max span)	4.92	1.2	2–8
Hearts and Flowers accuracy (%)	74.2	1.4	35–100
KeyMath 3 Numeration (scale score)	7.9	2.7	2–19
KeyMath 3 Algebra (scale score)	8.3	2.9	1–17
KeyMath 3 Geometry (scale score)	7.9	2.4	2–14
KeyMath 3 Composite (scale score)	8.0	2.4	2–16.3

Table 2

Bivariate correlations of cognitive and standardized test scores.

Measure (N = 472)	1	2	3	4	5	6	7	8
1. KeyMath 3 Composite								
2. KeyMath 3 Algebra	.94[**]							
3. KeyMath 3 Geometry	.87[**]	.69[**]						
4. KeyMath 3 Numeration	.94[**]	.84[**]	.72[**]					
5. Nonsymbolic Accuracy	.16[**]	.13[**]	.19[**]	.13[**]				
6. Symbolic Accuracy	.35[**]	.35[**]	.27[**]	.34[**]	.23[**]			
7. Mixed-format Accuracy	.14[**]	.15[**]	.10[*]	.14[**]	.13[*]	.25[**]		
8. Corsi Max Span	.38[**]	.36[**]	.32[**]	.36[**]	.12[**]	.22[**]	.10[*]	
9. Hearts & Flowers Accuracy	.39[**]	.37[**]	.35[**]	.36[**]	.21[**]	.33[**]	.14[**]	.21[**]

Variables 1–4 refer to age-scaled scores.

  * $p < 0.05$.
  ** $p < 0.01$.
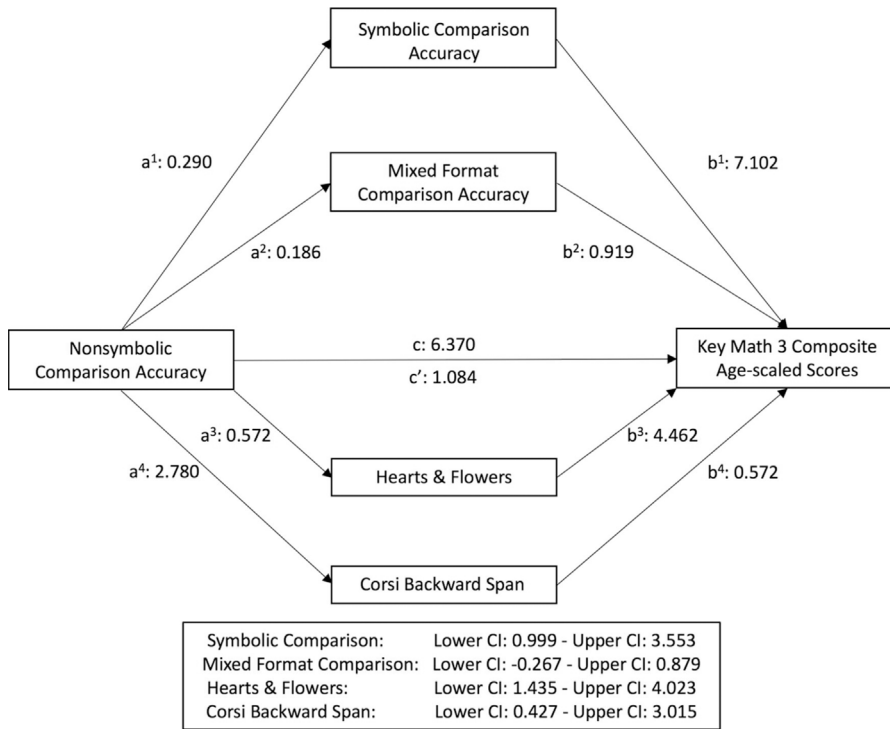
Fig. 2. Model 1: Mediation model showing the relations between nonsymbolic comparison and math performance with symbolic magnitude comparison, mixed-format comparison, hearts & flowers mixed block accuracy, and the backward Corsi max span as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

3.2.2. Mediators of the relation between symbolic magnitude processing and math

Recent evidence suggests there may be a bidirectional influence between nonsymbolic and symbolic representations of numerical magnitude, whereby the acquisition of symbolic number knowledge refines the representations in the ANS (Mussolin et al., 2015; Piazza et al., 2013). Therefore, we conducted a second mediation analysis in which symbolic magnitude processing was entered as the independent variable predicting KeyMath 3 composite with nonsymbolic magnitude processing as the proposed mediator, in addition to executive function measures (Hearts & Flowers and Backward Corsi). The confidence intervals resulting from this analysis did contain zero for nonsymbolic comparison performance, but not for executive function or working memory (Fig. 3), indicating that nonsymbolic magnitude comparison performance does not mediate the relation between symbolic magnitude comparison and math competence, while inhibitory control/task switching and working memory do. Further, similar to the results of model 1, performance on the mixed-format comparison task did not mediate the relation between symbolic comparison performance and math competence, suggesting that the relation is not dependent on efficiency of mapping between symbolic and nonsymbolic magnitude systems.

3.2.3. Executive function as control variables

Given that both measures of executive function mediated the relation between both nonsymbolic and symbolic comparison and
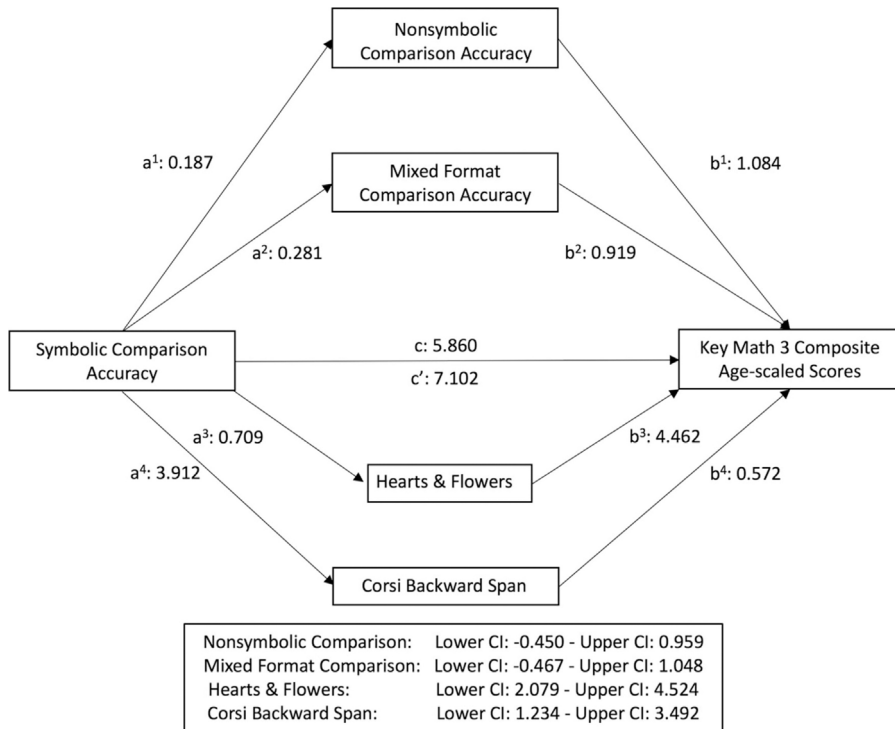
Fig. 3. Model 2: Mediation model showing the relations between symbolic comparison and math performance with nonsymbolic magnitude comparison, mixed format comparison, hearts and flowers mixed block accuracy, and Corsi max backward span as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

math, to assess the extent to which the mediating role of symbolic comparison was accounted for by those domain-general cognitive mechanisms, we replicated model 1 (i.e. symbolic comparison accuracy as the mediator) but included both measures of executive function as covariates instead of mediators. We did not perform this analysis for model 2 because nonsymbolic comparison was not a significant mediator in the original model. The results of this analysis continued to indicate full mediation for symbolic comparison (Lower CI = 0.602; Upper CI = 2.674). These results suggest that the mediating effect of symbolic magnitude processing on the relation between nonsymbolic processing and math pertains over and above the influence of inhibitory control/task switching and working memory.

3.2.4. Comparison of math outcome measures

Finally, to investigate whether the above relations differed as a function of math outcome, we replicated models 1 and 2 using KeyMath Geometry, Algebra, and Number subtests as dependent variables, as opposed to the composite math variable used in our main analyses. The results (Table 3) exactly mirror those when using the composite outcome measure, namely, symbolic comparison mediates the relation between nonsymbolic and math, but nonsymbolic does not mediate the relation between symbolic and math. In other words, there appears to be no difference between sub-tests.

3.3. Relations between nonsymbolic and symbolic magnitude processing, executive function, and mixed-format magnitude comparison

To investigate the cognitive mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing, we performed a mediation analysis with nonsymbolic comparison accuracy as the independent variable, symbolic comparison accuracy as the dependent variable, and mixed-format comparison accuracy and executive function measures (Hearts & Flowers and Backward Corsi) as the proposed mediators. Bias-corrected confidence intervals for the indirect effect in this model did not include zero for any of the mediators, indicating a mediating role for each (Figure 4).

While the executive function tasks used in this study are widely used indices of inhibitory control/task switching and working memory, the exact mechanism underlying performance on our mixed-format magnitude comparison task is less clear. Although we hypothesize that it indexes some degree of shared semantic representation between nonsymbolic and symbolic number formats, the fact that mixed comparison accuracy correlates with our measures of executive function leaves open the possibility that the mediating role of mixed-format comparison reflects the influence of domain-general, executive function factors shared between tasks, as opposed to a mechanism more specifically related to mapping between symbolic and nonsymbolic numerical magnitudes. To test this hypothesis, we performed an additional mediation analyses in which both executive function measures (Hearts & Flowers and Backward Corsi) were entered as covariates in the mediation model between nonsymbolic and symbolic comparison. Bias corrected

Table 3
Results of the mediation analyses for each KeyMath sub-test as a separate dependent variable.

Math Variable	IV	Mediator	IV to Mediator (a path)	Mediator to DV (b path)	Total Effect of IV on DV (c path)	Direct Effect of IV on DV (c' path)	Lower CI	Upper CI
Geometry	Nonsymbolic	Symbolic	0.290	4.488	4.924	3.607	0.396	2.616
		Mixed	0.186	0.092			−0.491	0.602
		H & F	0.572	3.997			1.238	3.729
		Corsi	2.780	0.476			0.369	2.568
Algebra	Nonsymbolic	Symbolic	0.290	8.719	7.417	−0.123	1.253	4.345
		Mixed	0.186	1.611			−0.198	1.172
		H & F	0.572	4.896			1.549	4.501
		Corsi	2.780	0.644			0.468	3.429
Numeration	Nonsymbolic	Symbolic	0.290	2.346	6.769	−0.232	1.163	4.032
		Mixed	0.186	0.197			−0.317	1.005
		H & F	0.572	0.699			1.378	4.140
		Corsi	2.780	0.695			0.429	3.184
Geometry	Symbolic	Nonsymbolic	0.187	3.607	5.393	4.488	−0.019	1.616
		Mixed	0.281	0.092			−0.728	0.776
		H & F	0.709	3.997			1.771	4.147
		Corsi	3.912	0.476			0.969	3.094
Algebra	Symbolic	Nonsymbolic	0.187	−0.123	6.419	8.719	−0.839	0.802
		Mixed	0.281	1.611			−0.387	1.432
		H & F	0.709	4.896			2.212	5.058
		Corsi	3.912	0.644			1.374	4.012
Numeration	Symbolic	Nonsymbolic	0.187	−0.232	5.769	8.098	−0.788	0.705
		Mixed	0.281	1.055			−0.535	1.238
		H & F	0.709	4.492			1.953	4.706
		Corsi	3.912	0.597			1.268	3.686
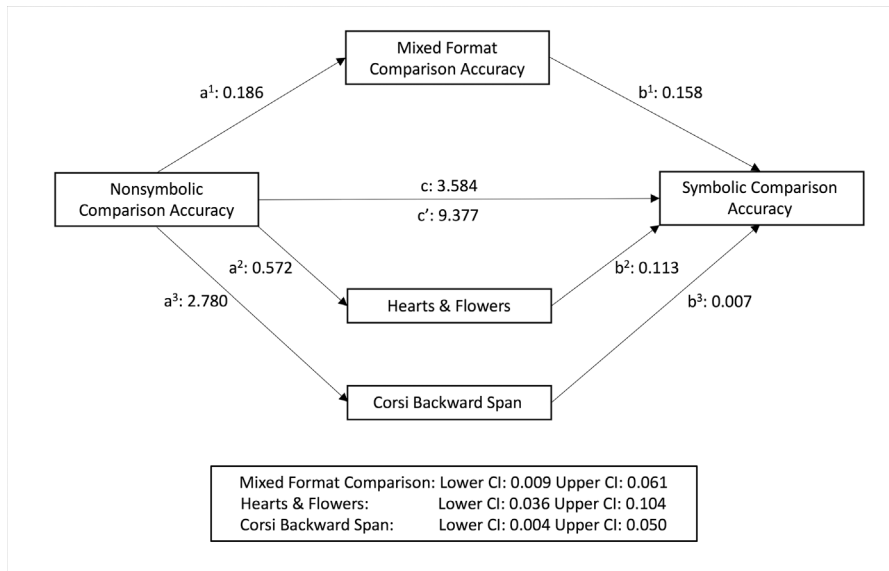
Fig. 4. Model 3: Mediation model showing the relations between nonsymbolic and symbolic magnitude comparison with mixed format comparison, hearts and flowers mixed block accuracy, and Corsi max backward span as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

confidence intervals for the indirect effect did not include zero when controlling for both measures of executive function (Lower CI = 0.003; Upper CI = 0.052), suggesting that mixed-format comparison accuracy fully mediates the relation between nonsymbolic and symbolic magnitude comparison over and above the effect of those domain-general cognitive mechanisms.

4. Discussion

4.1. Mechanisms underlying the relation between numerical magnitude processing and math

Several recent studies suggest that the relation between nonsymbolic magnitude processing skills and math competence is mediated by symbolic number processing skill (Fazio et al., 2014; Lyons & Beilock, 2011; Peng et al., 2017; Price & Fuchs, 2016; Wong et al., 2016). However, less is known about whether mapping between nonsymbolic and symbolic magnitude representations and the domain-general mechanisms of inhibitory control, task switching, and working memory also mediate that relation. To that end, in the current study we examine whether symbolic comparison, mixed-format comparison, and executive function each mediate the relation between nonsymbolic magnitude processing and math.

Our results indicate that symbolic magnitude comparison fully mediates the relation between nonsymbolic magnitude processing and math competence. These findings are consistent with those reported by Price and Fuchs (2016) for typically developing 3rd grade children, and are consistent with an emerging body of literature that suggests symbolic number processing more broadly, not just magnitude comparison, may mediate the influence of nonsymbolic magnitude processing on math development (Fazio et al., 2014; Lyons & Beilock, 2011).

Our results also indicate that inhibitory control/task switching (as measured in the Hearts and Flowers task) and working memory (as measured in the backward Corsi) each mediated the relation between nonsymbolic magnitude processing and math, as well as the relation between symbolic magnitude processing and math. A number of recent studies (Fuhs & McNeil, 2013; Gilmore et al., 2013) suggest that the relation between nonsymbolic magnitude processing and math may be accounted for by executive function processing related to processing numerical magnitude in the face of conflicting visual cues. The present results support existing findings by showing that multiple aspects of executive function mediate the relation between nonsymbolic magnitude processing and math, including inhibitory control.

Importantly, symbolic magnitude processing continued to mediate the relation between nonsymbolic magnitude processing and math when controlling for executive function and working memory, suggesting that the influence of symbolic magnitude processing goes beyond these domain-general cognitive mechanisms. This suggests that both domain-specific number processing and domain-general cognitive processes are involved in the scaffolding process from basic nonsymbolic magnitude processing to formal math competence. These results were consistent across the geometry, algebra, and number sub-tests of the KeyMath3 battery, suggesting that the observed relations hold true at a broad level and are not unique to specific subdomains of mathematics.

Interestingly, our results reveal that mixed-format comparison accuracy did not mediate the relations between nonsymbolic or symbolic comparison and math. If the mixed-format comparison task is taken to index the strength of mapping between nonsymbolic and symbolic representations of numerical magnitude, then these findings are in contrast to those of Wong et al. (2016), who found the number-numerosity mapping, as indexed by dot estimation, mediated the relation between nonsymbolic magnitude processing and math. The most likely explanation for the contradictory results lies in the differences between the 'mapping' tasks. The mixed-format comparison task employed in the current study does not require participants to generate a symbolic representation, but rather to compare two simultaneously presented stimuli. It is possible that the process of generating the symbolic output required in the estimation task employed by Wong et al., engages linguistic or verbal production processes beyond simply transcoding or 'mapping' between the two formats, and that those processes are pertinent to math development. Nonetheless, it is important to note that our findings suggest that although symbolic number processing mediates the relation between nonsymbolic magnitude processing and math, it is not the *mapping* between nonsymbolic and symbolic formats that underlies that relation. Therefore, some other aspect of symbolic number processing must account for the relation between nonsymbolic processing and math, and further research is required to determine exactly what that is.

It is also possible that results of the present study were influenced by the high number of children from low-income backgrounds included in the sample. Individuals from low SES backgrounds typically underachieve in math, with differences already evident in preschool (Sarama & Clements, 2009). A significant body of research suggests that the influence of SES is strongest on verbal and linguistic aspects of mathematics (for a review see Jordan & Levine, 2009), which may alter the influence of nonsymbolic-symbolic mapping processes. However, to fully investigate this possibility, the relation between basic measures of nonsymbolic and symbolic magnitude processing and math in low SES children needs to be empirically examined in contrast to a well-matched control group. Further research is clearly required to understand the source of the differences between the current findings and those of Wong et al.

Our results are also consistent with those reported by Price and Fuchs (2016) and Lyons and Beilock (2011) in that nonsymbolic magnitude processing did not mediate the relation between symbolic magnitude processing and math. Again, these results held true for each of the KeyMath sub-tests. While an emerging body of evidence suggests that the acquisition of symbolic number knowledge may lead to an increase in the precision of nonsymbolic magnitude representations (Mussolin et al., 2015; Piazza et al., 2013), the present results indicate a unidirectional influence from nonsymbolic through symbolic to math.

4.2. Mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing

The second principal aim of the current study was to assess whether the relation between nonsymbolic and symbolic magnitude processing is driven by a shared underlying representation of magnitude, or by shared domain-general cognitive mechanisms of executive function. While much of the extant literature assumes that overlapping performance profiles between nonsymbolic and symbolic magnitude comparison reflect the influence of a shared underlying representation of numerical magnitude, it is also possible that such overlap is the result of shared domain-general executive function mechanisms such as inhibitory control, task switching,

and working memory. The present study investigated this issue by assessing the extent to which the relation between nonsymbolic and symbolic magnitude comparison was mediated by mixed-format magnitude comparison and executive function. Our results demonstrated that mixed-format comparison, inhibitory control/task switching, and working memory mediated the relation between nonsymbolic and symbolic comparison, and importantly, that mixed-format comparison performance continued to mediate the relation between nonsymbolic and symbolic processing when controlling for executive function measures. These results suggest that the link between the two number formats is the product of both domain-general and domain-specific cognitive mechanisms, and that the mediating role of mixed-format comparison may, at least in part, reflect some degree of shared underlying representation of magnitude between the formats. An alternative explanation is that mixed-format comparison performance reflects the cognitive process of transcoding between numerical formats, and that more efficient transcoding ability enables better learning of numerical symbols. Given the limited literature on mixed-format comparison, and the fact that the current sample included a large proportion of children from low-income backgrounds, these interpretations require further empirical investigation. Importantly, the present results need to be replicated and with children from a full range of income backgrounds.

It should also be noted that, given the amount of assumed cognitive overlap between our three comparison conditions, the strength of statistical associations between them (nonsymbolic-symbolic $r = 0.23$, nonsymbolic-mixed $r = 0.13$, symbolic-mixed $r = 0.25$) were not as strong as might be intuitively expected. However, there are relatively few studies that have utilized the mixed-format experimental paradigm, and results are somewhat mixed. The first study to our knowledge to employ the mixed-format paradigm (Mundy & Gilmore, 2009) reported a lack of significant correlations among tasks in a group of 6- and 8-years-olds. Symbolic comparison accuracy rate correlated with mixed-format comparison at $r = -0.17$, *n.s.*, and nonsymbolic comparison accuracy rate correlated with mixed-format at $r = -0.04$, *n.s.* However, Brankaer, Ghesquière, and De Smedt (2014) reported significant correlation between accuracy rates for mixed-format and symbolic comparison of $r = 0.42$, and between mixed-format and nonsymbolic comparison ($r = 0.38$) in first- and third-graders. Lyons, Ansari, and Beilock (2012) did not report the correlations among these tasks, but do report that mixing symbolic and nonsymbolic representations comes at a significant cost for accuracy, indicating that across format comparisons require additional cognitive resources compared to within-format nonsymbolic or symbolic performance in a group of undergraduates. Therefore, our results fall directly between previously published results, albeit in a different age range, demonstrating a relatively weak but significant relationship. Given the lack of consistency in previous results and the fact that the age of our sample differs from the previous two studies, it remains an open question as to the degree of relation between tasks that require cross-format comparison vs. within-format comparison.

In summary, the present results suggest that the relation between nonsymbolic magnitude processing and math competence is mediated by both domain-specific symbolic magnitude processing, and by multiple components of domain-general executive function. This relationship appears to be unidirectional in that nonsymbolic magnitude processing does not mediate the relation between symbolic comparison and math. In contrast to a recent study, our results suggest that nonsymbolic-symbolic mapping does not mediate the relation between nonsymbolic or symbolic comparison and math. Finally, our results suggest that the relation between nonsymbolic and symbolic magnitude processing is accounted for by domain-general, executive function mechanisms as well as a domain-specific measure of nonsymbolic-symbolic mapping. The extent to which performance on the mixed-format task employed in the current study reflects *mapping* in the sense of a shared underlying representation of numerical magnitude across formats versus active transcoding processes, and the extent to which all of the present results generalize to samples from middle- and higher-income backgrounds requires further empirical investigation.

Acknowledgments

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cogdev.2017.09.003.

References

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development, 78*(2), 647–663. http://dx.doi.org/10.1111/j.1467-8624.2007.01019.x.

Blair, C., Knipe, H., & Gamson, D. (2008). Is there a role for executive functions in the development of mathematics ability? *Mind, Brain, and Education, 2*(2), 80–89. http://dx.doi.org/10.1111/j.1751-228X.2008.00036.x.

Brankaer, C., Ghesquière, P., & De Smedt, B. (2014). Children's mapping between non-symbolic and symbolic numerical magnitudes and its association with timed and untimed tests of mathematics achievement. *PUBLIC LIBRARY OF SCIENCE, 9*(4), e93565. http://dx.doi.org/10.1371/journal.pone.0093565.

Bugden, S., & Ansari, D. (2011). Individual differences in children's mathematical competence are related to the intentional but not automatic processing of Arabic numerals. *Cognition, 118*(1), 32–44. http://dx.doi.org/10.1016/j.cognition.2010.09.005.

Bynner, J., & Parsons, S. (1997). *Does numeracy matter?* London: The Basic Skills Agency.

Connolly, A. J. (2007). *KeyMath-3 diagnostic assessment.* San Antonio, TX: Pearson.

Corsi, P. (1972). Human memory and the medial temporal region of the brain. *Dissertation Abstracts International, 34*(2).

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology, 103*(4), 469–479. http://dx.doi.org/10.1016/j.jecp.2009.01.010.

De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education, 2*(2), 48–55. http://dx.doi.org/10.1016/j.tine.2013.06.001.

Dehaene, S. (2007). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Society, 970*(1999), http://dx.doi.org/10.1126/science.284.5416.970.

Diamond, A. (2014). Executive functions. *Annual Review of Clinical Psychology, 64*, 135–168. http://dx.doi.org/10.1146/annurev-psych-113011-143750 [Executive].

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123*(1), 53–72. http://dx.doi.org/10.1016/j.jecp.2014.01.013.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. http://dx.doi.org/10.1016/j.tics.2004.05.002.

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science, 16*(1), 136–148. http://dx.doi.org/10.1111/desc.12013.

Gebuis, T., & Reynvoet, B. (2011). Generating nonsymbolic number stimuli. *Behavior Research Methods, 43*(4), 981–986. http://dx.doi.org/10.3758/s13428-011-0097-5.

Gilmore, C., Attridge, N., & Inglis, M. (2011). Measuring the approximate number system. *Quarterly Journal of Experimental Psychology (2006), 64*(11), 2099–2109. http://dx.doi.org/10.1080/17470218.2011.574710.

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., ... Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PloS One, 8*(6), e67374. http://dx.doi.org/10.1371/journal.pone.0067374.

Gross, J., Hudson, C., & Price, D. (2009). *The long term costs of numeracy difficulties. London (UK): Every child a chance trust and KPMG (January 2009)*. Retrieved from http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:The+long+term+costs+of+numeracy+difficulties#0.

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455*(7213), 665–668.

Hayes, A. F. (2013). *Introduction to mediation, moderation, and conditional process analysis.* New York: Guilford Press.

Hofer, K. G., Lipsey, M. W., Dong, N., & Farran, D. C. (2013). *Results of the early math project – scale-up cross-site results.* Nashville,TN.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology, 103*(1), 17–29.

Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica, 145*, 147–155. http://dx.doi.org/10.1016/j.actpsy.2013.11.009.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*(December (2008)), 60–68. http://dx.doi.org/10.1002/ddrr.46.

Leibovich, T., & Ansari, D. (2016). The symbol-grounding problem in numerical cognition: A review of theory, evidence, and outstanding questions. *Canadian Journal of Experimental Psychology/Revue Canadienne De Psychologie Expérimentale, 70*(1), 12–23. http://dx.doi.org/10.1037/cep0000070.

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica, 141*(3), 373–379. http://dx.doi.org/10.1016/j.actpsy.2012.09.009.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition, 121*(2), 256–261. http://dx.doi.org/10.1016/j.cognition.2011.07.009.

Lyons, I. M., Ansari, D., & Beilock, S. L. (2012). Symbolic estrangement: Evidence against a strong association between numerical symbols and the quantities they represent. *Journal of Experimental Psychology. General, 141*(4), 635–641. http://dx.doi.org/10.1037/a0027248.

Lyons, I. M., Nuerk, H.-C., & Ansari, D. (2015). Rethinking the implications of numerical ratio effects for understanding the development of representational precision and numerical processing across formats. *Journal of Experimental Psychology. General, 144*(5), 1021–1035. http://dx.doi.org/10.1037/xge0000094.

Matejko, A. A., & Ansari, D. (2016). Trajectories of symbolic and nonsymbolic magnitude processing in the first year of formal schooling. *PLoS One, 11*(3), 1–15. http://dx.doi.org/10.1371/journal.pone.0149863.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011a). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development, 82*(4), 1224–1237. http://dx.doi.org/10.1111/j.1467-8624.2011.01608.x.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011b). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS One, 6*(9), e23749. http://dx.doi.org/10.1371/journal.pone.0023749.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex Frontal Lobe tasks: A latent variable analysis. *Cognitive Psychology, 41*(1), 49–100. http://dx.doi.org/10.1006/cogp.1999.0734.

Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology, 103*(4), 490–502. http://dx.doi.org/10.1016/j.jecp.2009.02.003.

Mussolin, C., Nys, J., Leybaert, J., & Content, A. (2015). How approximate and exact number skills are related to each other across development: A review☆. *Developmental Review,* 1–15. http://dx.doi.org/10.1016/j.dr.2014.11.001.

NCES (2007). *The Condition of Education.*

Odic, D., Hock, H., & Halberda, J. (2014). Hysteresis affects approximate number discrimination in young children. *Journal of Experimental Psychology. General, 143*(1), 255–265. http://dx.doi.org/10.1037/a0030825.

Parsons, S., & Bynner, J. (2005). *Does numeracy matter more? NRDC (National research and development centre for adult literacy and numeracy).* London: aRCK.

Peng, P., Yang, X., & Meng, X. (2017). The relation between approximate number system and early arithmetic: The mediation role of numerical knowledge. *Journal of Experimental Child Psychology, 157*, 111–124. http://dx.doi.org/10.1016/j.jecp.2016.12.011.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition.*

Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal approximate number system. *Psychological Science, 24*(6), 1037–1043. http://dx.doi.org/10.1177/0956797612464057.

Piazza, M. (2011). Neurocognitive start-up tools for symbolic number representations space. *Time and Number in the Brain, 14*(12), 267–285. http://dx.doi.org/10.1016/B978-0-12-385948-8.00017-7.

Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40*(3), 879–891. http://dx.doi.org/10.3758/BRM.40.3.879.

Price, G. R., & Fuchs, L. S. (2016). The mediating relation between symbolic and nonsymbolic foundations of math competence. *Plos One, 11*(2), e0148981. http://dx.doi.org/10.1371/journal.pone.0148981.

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). Nonsymbolic numerical magnitude comparison: Reliability and validity of different task variants and outcome measures, and their relationship to arithmetic achievement in adults. *Acta Psychologica, 140*(1), 50–57. http://dx.doi.org/10.1016/j.actpsy.2012.02.008.

Sarama, J., & Clements, D. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* New York: Routledge.

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *The British Journal of Developmental Psychology, 30*(Pt. (2)), 344–357. http://dx.doi.org/10.1111/j.2044-835X.2011.02048.x.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number-space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology, 114*(3), 418–431. http://dx.doi.org/10.1016/j.jecp.2012.10.012.

Wong, T. T., Ho, C. S., & Tang, J. (2016). The relation between ANS and symbolic arithmetic skills: The mediating role of number-numerosity mappings. *Contemporary Educational Psychology, 46*, 208–217. http://dx.doi.org/10.1016/j.cedpsych.2016.06.003.

Wright, A., & Diamond, A. (2014). An effect of inhibitory load in children while keeping working memory load constant. *Frontiers in Psychology, 5*(MAR), 1–9. http://dx.doi.org/10.3389/fpsyg.2014.00213.